DON'T WORK AROUND LANGUAGE IN MATH—DEVELOP IT. —Phil Daro

Integrating Language Development and Content Learning in Math: Focus on Reasoning

Franklin-McKinley
Jeffzwiers.org/june21

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WARM-UP - Transition Improv Activity

Topics: Addition-Subtraction, Area-Perimeter, mm-km
Multiplication-Division, Volume-Surface Area
Fractions-Decimals,

Transitions: **However,**
On the other hand,
Then again, ~~but~~

Frames: You need to add when … because…
When you … you need to subtract because…
You need to calculate volume when… because…
When you … you need to use decimals because...

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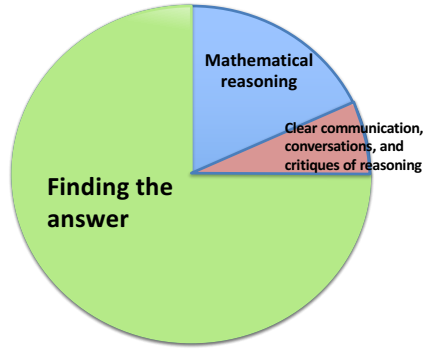

Today's Objectives

1. Develop practical ways to build a more communicative classroom

2. Improve at "squeezing out" as much reasoning and language as possible from each problem and activity (includes setting up, processing, and ending phases)

3. Develop teacher practices and activities that foster students' reasoning language in 3 modes: listening, speaking, and conversing

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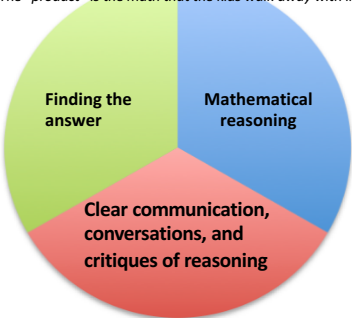

Previous Emphases

Mathematical reasoning

Clear communication, conversations, and critiques of reasoning

Finding the answer

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Target Emphases

Correct answers are essential… but they're part of the process, they're not the product. The "product" is the math that the kids walk away with in their heads.
—Phil Daro

Finding the answer

Mathematical reasoning

Clear communication, conversations, and critiques of reasoning

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Authentic Communication in School is

the use of words (and/or other meaning-carriers)
to share information
for doing meaningful things (creating, changing, deciding, clarifying, etc.)
that just one person can't do.
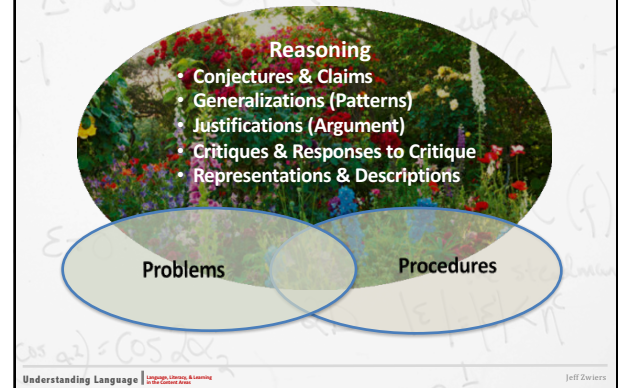
Why is it so important?

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6/10/17

Major Shift

"Instead of focusing on finding the answer, I often tell students that we are looking for new ways of solving the problem and the clearest ways to describe and justify them."

Key Genres of Math Language

Reasoning
• Conjectures & Claims
• Generalizations (Patterns)
• Justifications (Argument)
• Critiques & Responses to Critique
• Representations & Descriptions

Problems   Procedures

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Fostering a Mindset for Growing Mathematical Language: Communicativeness Features for Language Development

_____ **Is there a useful & engaging purpose?** In the activity, do students *use* (and *need to* use) language to do something meaningful and engaging beyond just to answer questions or get points? (e.g., language, content, thinking…) *Are there consequences for lack of clarity?*

_____ **Is there an information gap?** In the activity, do student get or give information that they want, need, or don't have?

_____ **Is there attention to language in service of communication?** In the activity, is there extra teaching and assessment focused on improving how language is used?

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning (CCSS College and Career Readiness Standards)
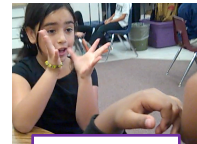
Which ones depend on reasoning?

Which ones require students to use academic language?

Foundational Principles
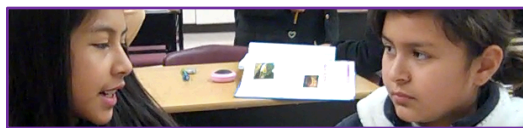
| Move all students forward (activities must allow & encourage this). | Language & content need & strengthen one another. | Meaningful communication accelerates & deepens learning of language & content | Describing, critiquing, & making sense of reasoning develops language & content |

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SPEAKING & LISTENING Activities

2

ACTIVITY:

Information Gap Activities

Purpose: To create a need for students to listen and talk. Students need to orally (and often visually) share their ideas and information in order to bridge the gap accomplish a task.

Info Gap Cards - Procedure

1. READ: A (Situation card) reads card, visualizes what is happening, and summarizes to B. B paraphrases the problem back to A, for agreement. Partner B reads the data card silently.
2. QUESTION 1: B asks, "What specific information do you need?" A needs to ask for specific information. "One thing I need to know is…"
3. QUESTION 2: Before answering, B should ask for justification: "**Why** do you need that information?"
4. EXPLANATIONS: A then explains how he or she will use the information to solve the problem. B can paraphrase or critique the justification. B asks for explanations, even if he or she understands what A is doing. Have them switch roles with new cards.
5. Have them switch roles with new cards.
6. FOLLOW-UP: As a follow-up step, have both students use blank cards to write their own similar problem card and data card for other pairs to use.
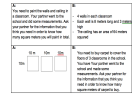
What are situations in which two people have to share math information?

Info Gap Activity: *Info Gap Cards*

A:
You need to paint the walls and ceiling in a classroom. Your partner went to the school and did some measurements. Ask your partner for the information that you think you need in order to know how many square meters you will paint in total.

B:
- 4 walls in each classroom
- Each wall is 8 meters long and 3 meters high
- The ceiling has an area of 64 meters squared

A: 8 m 10m 18m 15m
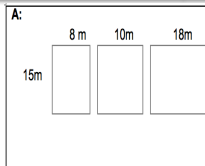
B:
You need to buy carpet to cover the floors of the classrooms in a small school. Your partner went to the school and made some measurements. Ask your partner for the information that you think you need in order to know how many square meters of carpet to buy.

Info Gap Activities
Info Gap Cards Sample Conversation

A: Do you know how fast the shuttle is orbiting?
B: Yes, but why do you want to know that?
A: Cuz I need to know it to figure out how long it takes to catch the satellite.
B: OK, it's going 16,800 miles per hour
A: Thanks. And how fast is the satellite going?
B: Why do you need to know that?
A: I need to know it to know how long it'll take. If it's just a little slower, it'll take longer. And what's the distance between them?
B: That makes sense. The satellite is going 16,000 mph
A: Thanks. And how far are they apart when the shuttle starts its orbit?
B: Why?

A: Model
A shuttle enters an orbital path to catch up to an important satellite that isn't working properly. The shuttle is going faster than the satellite and mission control wants to know when the shuttle will reach it.

B: Model
- Shuttle is orbiting at 16,800 mph
- Satellite orbits at 16,000 mph
- Shuttle enters orbit 1200 miles behind the satellite
- Orbit is 400 miles from the Earth's surface

Info Gap Cards: Your Turn

A:
Brendan and Shawn were out for a bike ride when τηε φαλλσ οφφ Βρενδανσ βικε, ωηιχη τηεψ λεαπε λοχκεδ υπ αρουνδ α τρεε. Τηεψ δεχιδε τηατ Βρενδαν ωιλλ βικε, φορ α ωηιλε ανδ Σηαων ωιλλ ριδε ηισ βικε, λεαπινγ ιτ φυρτηερ υπ τηε ροαδ ανδ ωαλκινγ τηε ρεστ οφ τηε ωαψ. Ωηεν Βρενδαν ρεαχηεσ Σηαων'σ βικε, ηε ωιλλ ριδε ιτ ηομε. Ηοω φαρ σηουλδ Σηαωδ τηε βικε φορ βοτη το αρριπε ηομε α σαμε τιμε?

B:
- The boys were πειντε κμ φρομ ηομε
- Βρενδαν ωαλκσ ατ χινθυε κμ/η ανδ ριδεσ ατ δοδιχηι κμ/η.
- Σηαων ωαλκσ ατ χετιρι κμ/η ανδ ριδεσ ατ διεχι κμ/η.

Info Gap Activities
Card Matching

Procedure

1. Model this several times.
2. Give story cards out to Student A in each pair, such as:
3. Have A read the card silently and picture what is happening.
4. Then A reads the card to B, who chooses from several graphs provided (or draws the graph).
5. B asks A for any information missed
6. A watches to help B, if needed
7. Students can also draw a graph and have the partner make up the story for it.

Elia walked away from her home. Then she realized that she forgot her lunch and ran toward home. Halfway back she decided to buy lunch, so she turned around to run to the bus stop and waited for the bus.

http://map.mathshell.org/lessons.php

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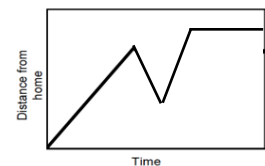
Info Gap Activities: Card Matching

STOP! How can we squeeze as much reasoning and language as possible out of this?

1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.
6. Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7. Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.
4. Tom has a mutation that allows him to be two places at once.

http://map.mathshell.org/lessons.php

Fostering a Mindset for Growing Mathematical Language: Essential Features of Communicativeness + SMPs

____ **Is there a useful & engaging purpose?** In the activity, do students *use* (and *need to* use) language to do something meaningful and engaging? *Are there consequences for lack of clarity?*

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____ **Is there attention to language in service of communication?** In the activity, is there extra teaching and assessment focused on improving how language is used?

Which SMPs are needed and emphasized (Focus, Abstract reasoning, Construct & critique arguments, Model, Tools, Precision, Structure, Regularity in repeated reasoning)

APPLY

Think about how you might use information gap activities and their features in your upcoming lessons.

ACTIVITY:

Stronger & Clearer Each Time Grid

Structured interaction for revising and clarifying ideas.

Stronger & Clearer Each Time Activities

1. Prompt for an **original response**
2. Successive partners: **borrow and use the language, ideas, and evidence** each time. Ideas become
   - **stronger** (often longer) with better
   - **clearer** with more precise terms and linked, organized, complete sentences.
3. Scaffolds **are reduced during** the activity.

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'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up to ounces of each thing.

Take one or two-word notes and switch partners!

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get, I don't know.

Pre-write:
Cut it up to ounces each bottle. Then add or times.
_ _ _ _ _ _ _ _ _ _ _ _
Post-write:

Silvia   1st Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain.

'Stronger & Clearer Each Time' Grid

I think to draw it. Then cut up to ounces of each thing.

Take notes & switch partners! Remember to say "because" to justify your steps

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get. I don't

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.

I wanna find how much a dollar can get, like of ounces. So 1 dollar is like 1 over 1.50, two thirds. So I take 2/3 of it?

Silvia

2nd Partner

Darla decides to buy a sports drink. Her choices are a 20-ounce bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain.
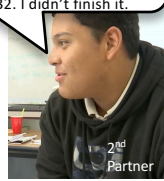
'Stronger & Clearer Each Time' Grid

Pre-write:
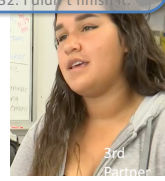
Cut it up to ounces each bottle. Then add or times it.
_ _ _ _ _ _ _ _ _ _ _ _
Post-write:

You need to find out how much each ounce costs. So I did cost over number of ounces. I got 7 for the 20 bottle.

I think we gotta find like how much ounces for a dollar it is. Like one dollar you get. I don't know.

I kinda did that, but I did for one ounce, its cost. I did 1.49 over 20. I think it's like 70 cents. And 32 over, no, 2.49 over 32. I didn't finish it.

First I thought to find how much ounces for a dollar. But then Alan gave me the idea to find each ounce, it costs. So I just do cost over the ounces. So like 1.49, divide 20 into it; Alan said 70 but I think its like 7.
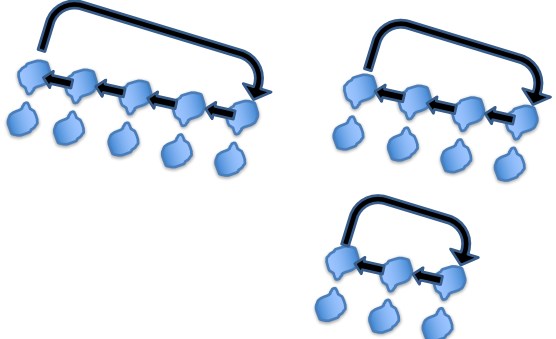
Silvia

3rd Partner

Darla decides to buy a sports drink. Her choices are a bottle for $1.49 or a 32-ounce bottle for $2.49. Which is the better value? Explain.

Practically Speaking: Interaction Mini-Lines

"Stronger & Clearer Each Time" Grid

Name	The school decided to put a rubber walkway of uniform width around the small swimming pool. The pool has a rectangular shape that measures 12 meters by 20 meters. The area of the walkway needs to be 144 m² because of the cost of the material. Find the maximum width of the walkway.	20 12
Me		
1.		
2.		*(You will present to the school principal on how to solve this problem.)*
3.		
Me		

We know…and this helps us to...
One constraint is...
I am not sure how to...but we might try...
I disagree with you...
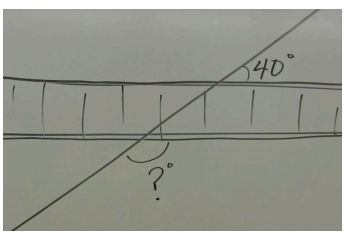
(Listeners can and should ask clarifying and supporting questions)

Video: 4th Grade Math

40°
?°

Context
- 4th grade Language math class in Redwood City
- Advanced and early advanced partners.

This Clip
- After working on a word problem on finding a supplementary angle, they met with three partners
- Ana practices describing her ideas; watch if and how her idea evolves

Looking at Student Work (Before & After Grid Partners)

EssentialQuestion: How do you find a solution to a system of equations when both equations are in standard form?

First Attempt
System is 2 equation I know that at the end of the steps your answer needs to be in co-ordinate pairs.

Stronger
$x - 3y = 14$ $2 - 3y = 14$
$x - 2 = 0$ $-7 - 2$
-34 -12
$(2, 4)$ -3
$y = 4$
At least one variable needs to be isolated. Replace (plug in) solve. Answer probmes co-ordinate pair.

Clearer
Make sure 1 variable is isolated & it has to be x's much. After that plug in using parenthesis. When you solve it it. To get out of the parenthesis use multiplication. then like terms are combined. divide the x and the number. You now need to find y. choose any equation and plug in. Then once you get your answer you write it in co-ordinate pair.

Fostering a Mindset for Growing Mathematical Language:
Essential Features of Communicativeness + SMPs

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MATH CONVERSATION Activities

ACTIVITY:
Paired Conversation Protocol

To support and scaffold productive student conversation of students who are working in pairs and jointly solving a problem with more than one solution method.

Math Paired Conversation Protocol

PROBLEM:

Paraphrase and clarify problem for one another (in pairs)
(Talk about what is asked; what is given; what happens; what the units are, etc.)
☐ *TALK*

Estimate the answer
(Each partner generate and justify your own estimate; then compare them)
☐ *TALK*

METHOD A *(name it)*	Justify method ☐*TALK*	**METHOD B** *(name it)*	Justify method ☐*TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do
	☐*TALK*		☐*TALK*

Math Paired Conversation Protocol

Check answer and compare to estimated ones ☐*TALK*	Check answer and compare to estimated ones ☐*TALK*
Discuss (argue) which method you would recommend for problems like this. Why? ☐*TALK*	
Discuss connections between the two methods. How do they relate? ☐*TALK*	
Generate a final explanation for how to solve problems like this; use this problem as an example. ☐*TALK*	
Co-create a similar problem, write it on the back of this sheet, and solve it (then share the problem with others) ☐ *TALK & WRITE*	

Math Paired Conversation Protocol

Suppose it takes the Almond River 6 years to fill a reservoir and Campbell Creek 10 years to fill it. If both are flowing into the reservoir, how long will it take to fill it?

PROBLEM:

Paraphrase and clarify problem for one another (in pairs)
(Talk about what is asked; what is given; what happens; what the units are, etc.)
☐ *TALK*

Estimate the answer
(Each partner generate and justify your own estimate; then compare them)
☐ *TALK*

METHOD A *(name it)*	Justify method ☐*TALK*	**METHOD B** *(name it)*	Justify method ☐*TALK*
Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do	Visuals, Drawings, Charts, Symbols, Calculations, Solution	Justify what you do
	☐*TALK*		☐*TALK*

Sample Conversation Using the

A: What do we gotta find?
B: How long they take to fill the reserve.
A: I say less than 6.
B: Why?
A: The Almond takes 6 months itself.
   So with extra water from this other one, less time, right?
B: Maybe. So we can't average 'em. So, maybe we draw it for one way to solve.
A: So like two rivers into a tank, like a box?
B: Yeah, and it fills up. After 3 months it's half full from Almond, right? But Belfair only fills up like, what?
A: 3 out of 10 is, three 10ths of it full on that side.
B: So, not full. So let's just guess it. Like I say/
A: /We can't do that. I think there's a right answer.
B: OK, let's try the other way, like a graph or a table.

> Suppose it takes the Almond River 6 months to fill a reservoir, by itself, and it takes Belfair River 10 months to fill it, on its own. If both are flowing into the reservoir, how long will it take to fill it?

Analyze a Conversation Sample

A: What do we need to find?
B: How far the boat goes down the river.
A: So, how?
B: Maybe figure out the time to cross it, like straight, like this (a).
A: I think we should just add the speeds together.
B: OK, that's 5 plus 3 equals 8. Then what?
A: We need to use the other number, 30. So divide?
B: Why not. OK, so 30 divided by 8 is 3.75.
A: 3.75 what?
B: Meters, I think, but that doesn't look right.
A: No, so what do we do?
B: I don't know.
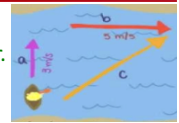
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APPLY

Think about how you might use the paired conversation protocol in your upcoming lessons.

Sample Lesson Plan: Math

I.   **Objective:** Collaborate to solve word problems with multiple solution methods

II. **Launch Problem in Pairs:** Pairs try to talk through ways to solve a new problem (find the price point), with whole group sharing of strategies and questions.

III. **Stronger & Clearer:** Students meet with successive partners to improve, clarify, and expand their solution ideas

IV. **Info Gap Problem:** A has situation; B has data

V.   **Math Paired Conversation Protocol:** Emphasize skills of supporting solution ideas with the words of the problem and math principles.

VI. **Co-Crafting Conversations:** Co-Write a similar but more challenging word problem.

Using the Instructional Design Tool
for
Developing Math Language

1. Consider content & skills to be learned (and how they will be assessed):	2. Reasoning emphasis: **Conjecture Generalize Justify Critique Represent/ Describe**	3. Choose a mode or 2 to emphasize **LISTEN SPEAK CLASS DISCUSSION CONVERSE WRITE/ DRAW**	4. Choose, Adapt, Design ACTIVITYs:	5. Strengthen its communicativeness • Useful & engaging **purpose** • **Information gaps** • **Attention** to language in service of communication

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1. Consider Content & Practices to be Learned

1. Consider math content & practices to be learned (and how they will be assessed:

A lesson is an opportunity to advance students' widely varying understandings, practices, language, and skills a bit further than they had the day before.

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2. Choose a Reasoning Emphasis

2. Reasoning emphasis:

Conjecture

Generalize

Justify

Critique

Describe/ Represent

Use your observations of classroom interactions, analyses of student work, and analyses of the assessments to choose one or two of these.

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3. Choose 1 or 2 Language Modes to Emphasize:

LISTEN

SPEAK

WRITE/DRAW

CLASSROOM DISCUSSION

CONVERSE

Cultivate a Culture of Communication

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4. Choose, Adapt, Design an Activity(s)

Consider the first three dimensions, your students' needs, and curriculum resources...

4. Choose,	Activities that Foster REASONING & Its Language (Conjectures, Claims, Generalizations, Justifications, Critiques, Descriptions, Representations)
Listening & Speaking	• **Stronger & Clearer Each Time Grid** • **Information Gap A & B Forms & Cards** • **Critique a Flawed or Partial Response** (Oral) • **Compare and Connect** Descriptions of Thinking by Peers • Gallery Walk • Think-Pair-Share
Class Discussion	• **Whole Group Discussion Supports** • Central Focus Semantic Map • Note-Taking Supports
Conversing	• **Paired Conversation Protocol** • **Sometimes, Always, Never** • Conversation Skills Poster • Sentence Frames
Writing/ Drawing	• **Co-create a problem or question** • **Critique a Flawed or Partial Response** (Written) • **Stronger-Clearer Grid** (Pre- and Post- Written Description) • Gallery Walk

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5. Strengthen Features of Communicativeness

Is there a useful & engaging purpose? In the activity, do students use language to do something meaningful and engaging (beyond just to answer questions or get points)?

Is there an information gap? In the activity, do students get or give information that they want, need, or don't have? Do they try to understand one another? + Are there consequences for lack of clarity?

Is there attention to language in service of communication? In the activity, is there extra teaching and assessment focused on improving how language is used? Includes modeling, scaffolding, practicing, giving feedback.

5. Strengthen its communicativeness

• Useful & engaging **purpose**
• **Information gaps**
• **Attention** to language in service of communication

Instructional Design Tool *for* **Developing Math Language**

1. Consider content & skills to be learned (and how they will be assessed):	2. Reasoning emphasis: Conjecture Generalize Justify Critique Represent/Describe	3. Choose a mode or 2 to emphasize LISTEN SPEAK CLASS DISCUSSION CONVERSE WRITE/DRAW	4. Choose, Adapt, Design ACTIVITYs:	5. Strengthen its communicativeness • Useful & engaging purpose • Information gaps • Attention to language in service of communication

Learn how to divide fractions	Describe when, why, & how	Speaking – use >1 sentence + Refer to problem and to math principles	Gallery Walk	More engaging purpose, Different problems More modeling

Instructional Design Tool *for* **Developing Math Language**

1. Consider content & skills to be learned (and how they will be assessed):	2. Reasoning emphasis: Conjecture Generalize Justify Critique Represent/Describe	3. Choose a mode or 2 to emphasize LISTEN SPEAK CLASS DISCUSSION CONVERSE WRITE/DRAW	4. Choose, Adapt, Design ACTIVITYs:	5. Strengthen its communicativeness • Useful & engaging purpose • Information gaps • Attention to language in service of communication

Next Steps