Grade 3: Properties of Multiplication and Division

Level 1:
I can understand *equal groups of* as multiplication and can relate multiplication to the array model

Level 2:
Interpret the meaning of factors- the size of the group or the number of groups

Level 3:
I can complete division problems (units 2-5 & 10) and find an unknown factor

Level 4:
I can interpret & explain the quotient as the number of groups or the number of objects in each group

LT#3 (3.OA.3, 3.OA.5, 3.OA.7, 3.OA.8):
 Applying tools & representations to problem-solving with multi-step word problems

Level 1:
I can identify the two steps (hidden question, addition problems) in a word problem

Level 2:
I can solve two- step word problems involving multiplication and division

Level 3:
I can solve two-step word problems involving all four operations
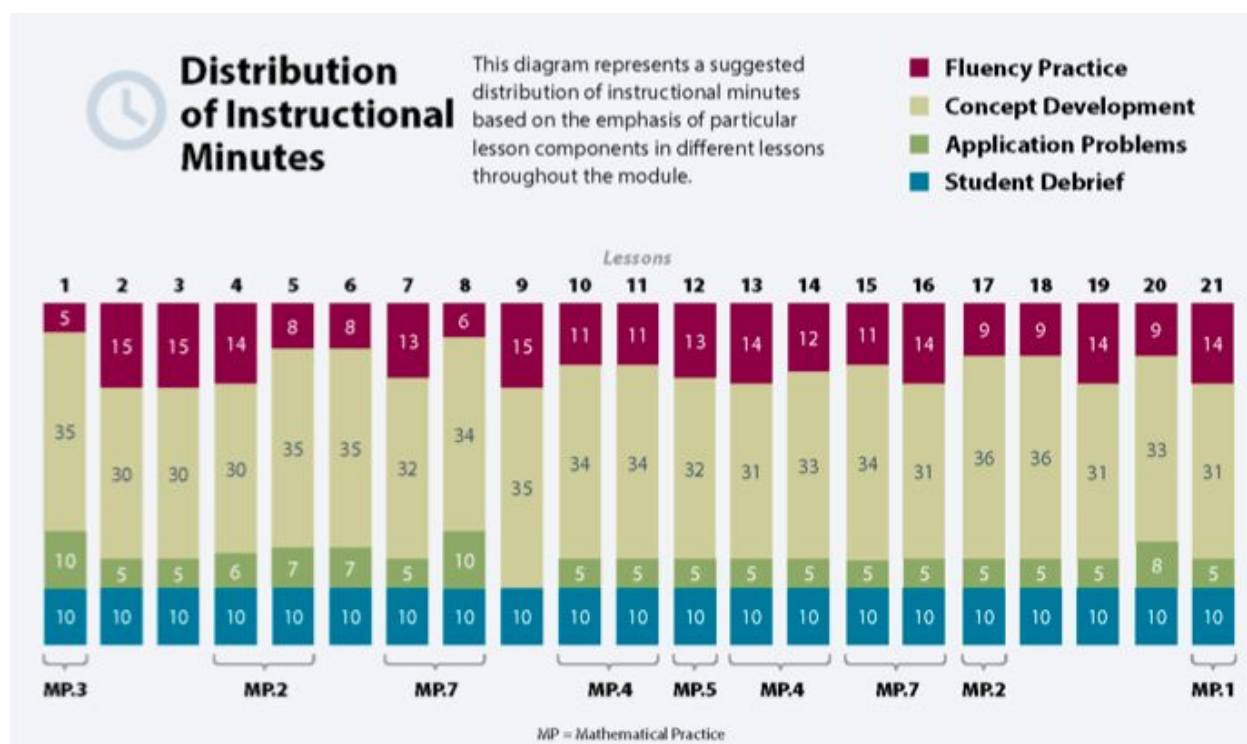
Level 4:
I can assess the reasonableness of my answers using a number of strategies (i.e. checking work, reverse operations, etc

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Consolidate Lessons 12 and 13, both of which are division lessons sharing the same objective. Include units of 2 and units of 3 in the consolidated lesson.

Omit Lessons 15 and 19. Lesson 15 uses the tape diagram to provide a new perspective on the commutative property, a concept students have studied since Lesson 7. Lesson 19 introduces the significant complexity of the distributive property with division. The concepts from both lessons are reinforced within Module 3.

Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

Legend:
- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief

Lessons

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
5	15	15	14	8	8	13	6	15	11	11	13	14	12	11	14	9	9	14	9	14
35	30	30	30	35	35	32	34	35	34	34	32	31	33	34	31	36	36	31	33	31
10	5	5	6	7	7	5	10		5	5	5	5	5	5	5	5	5	5	8	5
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

MP.3   MP.2   MP.7   MP.4   MP.5   MP.4   MP.7   MP.2   MP.1

MP = Mathematical Practice

Focus Grade Level Standards
Represent and solve problems involving multiplication and division.[1]

3.OA.1   Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.2   Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3   Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4   Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Understand properties of multiplication and the relationship between multiplication and division.[2]

3.OA.5   Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then*

*15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)***[3]**

3.OA.6   Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.**[4]**

3.OA.7   Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[5]**

> **3.OA.8**   Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Focus Standards for Mathematical Practice

> **MP.1**   **Make sense of problems and persevere in solving them.** Students model multiplication and division using the array model. They solve two-step mixed word problems and assess the reasonableness of their solutions.

& Solving Problems w/units of 2,3,4,5, &10

Learning Targets for Grade 3 Module 1 Engage NY

LT#1 (3 OA.1, 3.OA.4, 3.OA.5, 3.OA.7):
 Understand properties of multiplication & represent and solve multiplication problems.

Level 1:
I can understand *equal groups of* as multiplication and can relate multiplication to the array model

Level 2:
Interpret the meaning of factors- the size of the group or the number of groups

Level 3:
I can complete multiplication problems (units 2-5, & 10) and explain the meaning of factors

Level 4:
I can model & explain distributive property with arrays to decompose units as a strategy to multiply.

LT#2 (3.OA.2, 3.OA.4, 3.OA.6, 3.OA.7):

Understand properties of division & represent and solve division problems.

Level 1:
I can understand the meaning of the unknown as the size of the group in division and can interpret the unknown in division using the array model

MP.2	**Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they explore the properties of multiplication and division and the relationship between them. Students decontextualize when representing equal group situations as multiplication and when they represent division as partitioning objects into equal shares or as unknown factor problems. Students contextualize when they consider the value of units and understand the meaning of the quantities as they compute.
MP.3	**Construct viable arguments and critique the reasoning of others.** Students represent and solve multiplication and division problems using arrays and equations. As they compare methods, they construct arguments and critique the reasoning of others. This practice is particularly exemplified in daily Application Problems and in specific lessons dedicated to problem solving in which students solve and reason with others about their work.
MP.4	**Model with mathematics.** Students represent equal groups using arrays and equations to multiply, divide, add, and subtract.
MP.7	**Look for and make use of structure.** Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division equation.

---

[1]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[2]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[3]The associative property is addressed in Module 3.

[4]Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[5]In this module, problem solving is limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Learning Targets for Grade 4 Module 1 EngageNY
Place Value, Rounding, and Algorithms for Addition and Subtraction

Total Number of Lessons to be Covered: 10 sessions
Assessments: 2 sessions (mid and summative)

LT #1 (4.NBT.1, 4.NBT.2, 4.OA.1)
Understand place value of whole numbers less than or equal to 1,000,000.

Level 1:
I can correctly draw a place value chart and show a multidigit number up to 1 million.

Level 2:
I can use my model to explain why each digit of a number has a different value and can correctly indicate its value.

Level 3:
I can explain in words why each digit of a number has a different value.

Level 4:
I can read and write multidigit numbers in word form, standard form, and expanded form.

Notes: Complete Lessons 3 & 4 and skip Lessons 1 & 2.

LT #2 (4.NBT.2)
Compare multidigit whole numbers.

Level 1
I can use the place value chart to compare the value of each digit in order to determine which number is of greater value.

Level 2
I can compare two numbers by observing across the entire number and notice value differences.

Level 3

I can arrange whole numbers up to 1 million from least to greatest or greatest to least.

Level 4

I can create two different five-digit numbers and compare the two numbers.

Notes: Complete Lesson 6 and skip Lesson 5.

LT #3 (4.NBT.3)

Round multidigit whole numbers.

Level 1

I can round to the nearest thousand using a vertical number line.

Level 2

I can round to any given place value using a number line.

Level 3

Estimate sums and differences by rounding the appropriate numbers to an indicated place value.

Level 4

I can solve complex and real world examples of rounding (rounding up and rounding down).

Notes: Complete Lessons 7 and 8 and either 9 or 10.

LT #4 (4.OA.3, 4.NBT.4, 4.NBT.1, 4.NBT.2)

Add multidigit whole numbers and apply in appropriate problem situations.

Level 1

I can add multidigit whole numbers using special strategies.

Level 2

I can add multidigit whole numbers using the standard addition algorithm with and without regrouping.

Level 3

I can apply the addition algorithm within the context of word problems and assess the reasonableness of my answers using rounding.

Level 4

I can solve addition word problems using tape diagrams to model the situations and variables to represent the unknown quantities.

<u>Notes: Complete Lessons 11 and 12 and skip Lessons 17, 18, and 19.</u>

Lt #5 (4.OA.3, 4.NBT.4, 4.NBT.1, 4.NBT.2)
Subtract multidigit whole numbers and apply in appropriate problem situations.

Level 1
I can subtract multidigit whole numbers using special strategies.

Level 2
I can subtract multidigit whole numbers using the standard addition algorithm involving only 1 decomposition.

Level 3
I can subtract multidigit whole numbers using the standard addition algorithm involving multiple decompositions.

Level 4
I can solve subtraction word problems using the strategies I learned in addition and assess the reasonableness of my answers using rounding.

<u>Notes: Complete Lessons 13, 15, and 16 and skip Lessons 17, 18, and 19.</u>

Learning Targets for Grade 5 Module 1 EngageNY
Place Value and Decimal Fractions

LT #1 (5.NBT.1)
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Level 1:
I can correctly draw a place value chart and show a multi-digit number from millions to thousandths.

Level 2:
I can use my model to explain why each digit of a number has a different value and can correctly indicate its value.

Level 3:
I can explain in words why each digit of a number has a different value.

Level 4:
I can explain why each digit in a number has a different value by using "1/10 as large as" and "10 times as large as" for adjacent digits.

LT #2 (5.NBT.2)
Explain patterns in the number of zeroes of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Level 1
I can fluently move from a verbal to a numerical to an exponential representation of 10 to a whole-number exponent.

Level 2
I can determine the correct number of zeroes in a product when multiplying a decimal number by a power of 10.

Level 3

I can determine the correct answer when multiplying or dividing a decimal number by a power of 10.

Level 4
I can explain why a decimal point is placed in a particular position when multiplying or dividing a decimal number by a power of 10.

LT #3 (5.NBT.3)
Read, write and compare decimals to thousandths.

Level 1
I can compare two decimals to thousandths in numeral form.

Level 2
I can read and write decimals to thousandths in numeral form, using number names, and in expanded form.

Level 3
I can fluently change decimals among the three representations: base-ten numeral form, number names, and expanded form.

Level 4
I can order decimals to thousandths on a number line.

LT #4 (5.NBT.4)
Use place value understanding to round decimals to any place.

Level 1
I can round a decimal number in hundredths to the nearest tenth.

Level 2
I can round a decimal number in thousandths to the nearest tenth.

Level 3
I can estimate an answer by rounding decimals to the nearest tenth and then doing the indicated operation.

Level 4
I can analyze a verbal situation in which one has to round decimals to the nearest tenth and then decide which operation to use to answer the question posed.

Lt #5 (5.NBT.7)
Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Level 1
I can accurately add, subtract, multiply and divide decimals to hundredths using a numerical algorithm.

Level 2
I can add, subtract, multiply and divide decimals to hundredths, using concrete materials or a drawing.

Level 3
I can use the distributive property to show all the partial products in an area model for multiplication with decimals through hundredths. I can explain how this relates to a standard algorithm.

Level 4
I can use properties of multiplication and division by powers of ten to quickly and efficiently solve number strings involving decimals to hundredths.

LT #5 (5. MD.1)
Convert among different-sized standard measurement units with a given measure system and use these conversions in solving multi-step, real world problems.

Level 1
I can apply operations on decimals to hundredths from a verbal problem when the operation to be performed is specified.

Level 2
I can use a result from one part of a multi-step problem to figure out the answer to another part of the problem.

Level 3
I can analyze a verbal problem to determine which operation, addition, subtraction, multiplication or division, is appropriate to answer the question(s) posed.

Level 4
I can correctly answer all parts to a multi-step verbal problem, and explain strategies used to solve the problem.

Grade 6 Learning Targets for CPM Course 1 Chapter 2

LT #1: (6.SP.4)
Display numerical data in plots on a number line, including dot plots, histograms, and box plots **Venn Diagrams**.

(Note that the CPM text includes Venn Diagrams but does not discuss box plots in this chapter.)

Level 1:
I can construct a correct dot plot, bar graph, or Venn Diagram for a given set of data. I can answer questions about the data set by interpreting these types of graphs.

Level 2:
I can analyze a set of data and determine whether a dot plot, bar graph, or Venn Diagram would be best suited to display the data. I can justify my reasoning.

Level 3:
I can explain the similarities and differences between a histogram and a bar graph, and describe when each is more appropriate. I can answer questions about a data set when given a histogram of the data.

Level 4:
I can create a histogram to represent a set of data. I can justify my choice of bin size, and explain how changing the bin size will impact the overall shape of the graph.

LT #2: (6.G.1)
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Level 1:
I can determine the area of a shape by placing pattern blocks, when different blocks represent 1 unit of area.

Level 2:
I can describe what "one square unit" means. I can draw examples of specific square units, using proper notation. I can determine the area of a rectangle by counting the number of square units that completely fill the rectangle.

Level 3:

I can determine the area of a rectangle by drawing a sketch and determining the number of square units (without counting).

Level 4:

I can find the area of an unfamiliar or non-standard shape by breaking it into square units. Depending on the given information, I can either find an exact area or an appropriate estimate.

LT #3:

I can determine the perimeter and area of a shape (that can be decomposed into rectangles), and I can describe the relationship between area and perimeter.

Level 1:

I can use base-10 blocks to build or draw a shape that has a specified area. I can find the perimeter of the shape I created.

Level 2:

I can find the area and perimeter of a shape by first breaking it into non-overlapping rectangles.

Level 3:

I can use base-10 blocks to build or draw a shape that has a specified area *multiple different ways*, and compare their perimeters.

Level 4:

When given a particular area, I can determine whether it can be made into a rectangle where neither side length is 1, and whether it can be made into a square. I can justify my reasoning.

LT #4: (6.NS.4)

Find the greatest common factor of two whole numbers less than or equal to 100 <u>and the least common multiple of two whole numbers less than or equal to 12</u>. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Level 1:

I can explain what a factor is, and I can determine the GCF of two numbers.

Level 2:

I can represent the area of a generic rectangle (like the one shown) with either an addition statement, or a multiplication statement, and demonstrate their equivalence.

Level 3:

I can write an equation that represents area (as in level 2), where the factor involved is the GCF of the two numbers.

Level 4:
I can rewrite a multiplication problem by breaking one of the factors into two terms, draw a generic rectangle to represent it, and then write an equation that represents the area (as in level 2).

Example: 5 • 13 à 5(10 + 3) à   à   à   5(13) = 5(10 + 3) = 50 + 15

LT #5: (6.EE.3)
I can represent multi-digit multiplication problems with generic rectangles.
(Apply the properties of operations to generate equivalent expressions.)

Level 1:
Given a generic rectangle with all of the side lengths labeled, I can find the area of each smaller rectangle, and add them up to find the area of the whole rectangle. I can then use this diagram to write a correct multiplication statement.

Level 2:
I can decompose each factor of a multiplication problem into terms (by place value), draw a generic rectangle to represent it, and then find the area of each smaller rectangle. I can show that the sum of these smaller rectangles is equal to the product of the original numbers.

Level 3:
I can find missing areas and side lengths in a generic rectangle puzzle, and then write a number sentence showing that the sum of the smaller rectangles is equal to the product of the total side lengths.

Level 4:

I can write a short paragraph explaining how to use generic rectangles to multiply two 3-digit numbers without a calculator.

LT #6: (6.EE.3)

I can use the distributive property to rewrite products as sums.

(Apply the properties of operations to generate equivalent expressions.)

Level 1:

I can use the distributive property to change a numeric expression of the form into the form .

Level 2:

I can rewrite a multiplication problem (1-digit • 2-digits) by breaking the 2-digit number into two terms. I can then apply the distributive property to rewrite the product as a sum.

Level 3:

I can rewrite a multiplication problem (2-digits • 2-digits) by breaking each number into two terms. I can then apply the distributive property twice to rewrite the product as a sum.

Level 4:

I can write a short paragraph that defines what the distributive property is, and how it can be used to change products of multi-digit numbers into sums.