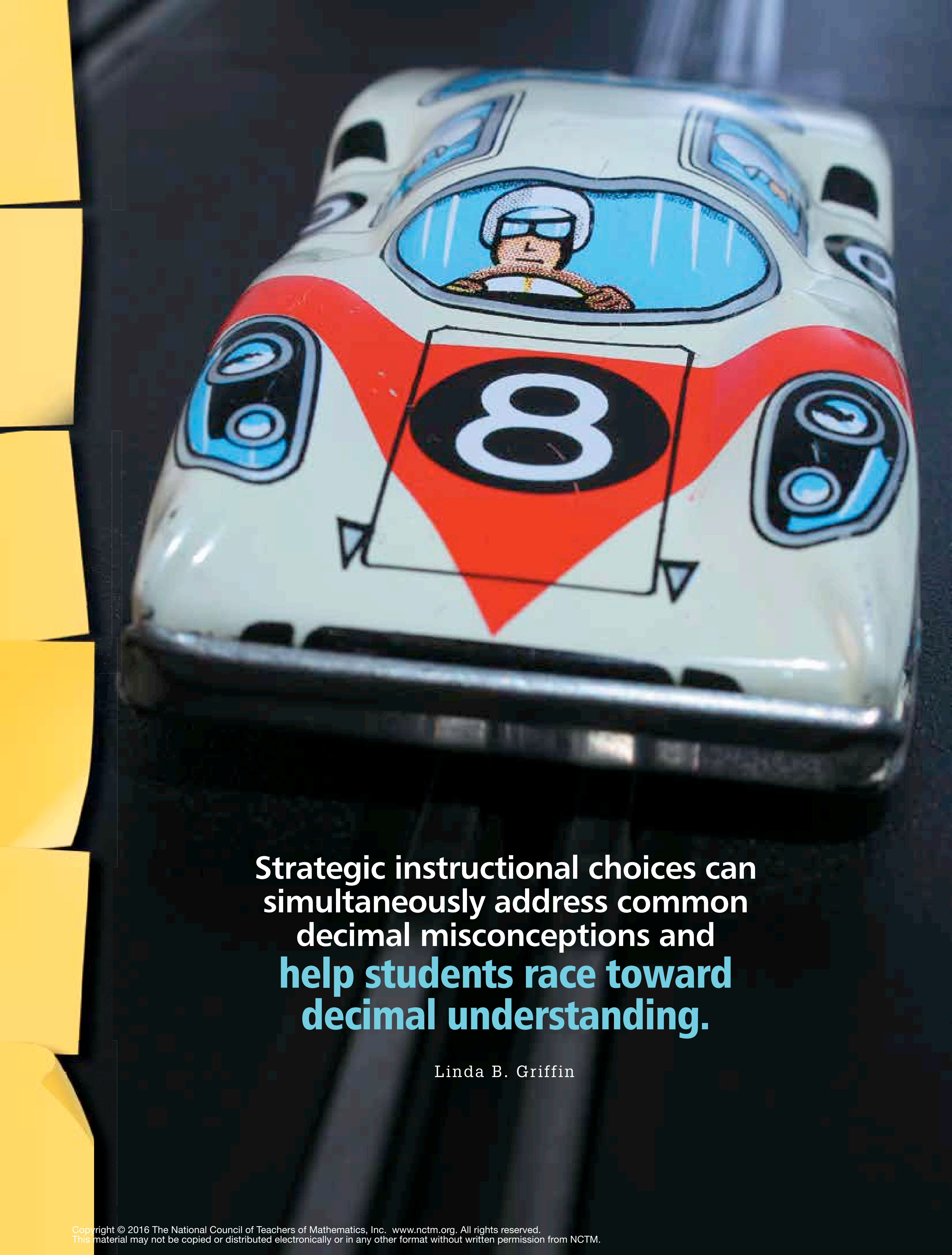


Strategic instructional choices can simultaneously address common decimal misconceptions and help students race toward decimal understanding.

Linda B. Griffin

Tracking Decimal Misconceptions:

STRATEGIC INSTRUCTIONAL CHOICES

Consider the student remarks below, overheard in a fourth-grade classroom:

Tala: Decimals are like fractions, so 3/4 = 0.34, right?

Carrie: Of course, 4.79 is greater than 4.8 because 79 is way more than 8.

Xander: Decimals get smaller and smaller the farther you go to the right of the decimal point. That's why 0.431 is smaller than 0.4.

Many teachers encounter these and other decimal misconceptions expressed by students. Tala's fraction rule, Carrie's longer-is-larger generalization, and Xander's longer-is-smaller theory are common reasoning errors (Bamberger, Oberdorf, and Schultz-Ferrell 2010; Martinie 2007; Stacey, Moloney, and Steinle 2004). It is easy to see why confusion emerges. Understanding the decimal system is challenging, requiring coordination of place-value concepts with features of whole-number and fraction knowledge (Moloney and Stacey 1997). Moreover, the learner must discern if and how previously learned concepts and procedures apply. The process is complex, and misconceptions will naturally arise. In a constructivist learning environment, teachers encourage student conjectures because errors and incomplete understandings often present powerful learning opportunities (Bamberger, Oberdorf, and Schultz-Ferrell 2010). However, when ignored or inadvertently reinforced, misconceptions can impede future learning (Moloney and Stacey 1997). Strategic instructional choices help students develop understanding and address potential misconceptions.

Understanding decimal notation and making decimal comparisons are central concepts in fourth grade (CCSSI 2010). To help her students achieve these goals, Ms. Campos makes several key decisions in the following lesson sequence, starting with her choice of materials. Rather than using prepartitioned item, such as base-ten pieces, hundred grids, or metersticks, Campos uses a minimally labeled strip of adding machine tape. Students will partition their strips into successively smaller decimal units, thereby reinforcing the multiplicative structure of the decimal system and proactively addressing several decimal misconceptions.

Day 1

Objective

Name positions on the number line using decimals (in tenths); compare and order tenths values between 0 and 2.

Launch: Making connections

Campos starts with a question for her fourth graders: "What do you already know about decimals?"

Her students tell her that decimals are numbers with a point, are related to fractions, and are seen in some familiar applications: money, metric measurements, and radio station frequencies.

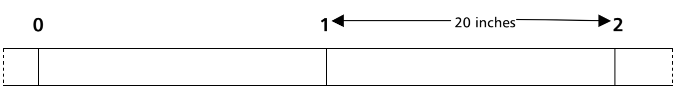
During the lesson launch, Campos makes a language connection: "What is the link between these words: *decade*, *decagon*, *decathlon*, and *decimeter*?"

FIGURE 1

Students created decimal number lines to model a miniature two-mile racetrack with a one-mile interval on the racetrack divided into ten equal parts.

Advance preparation:

Adding machine tape strip, approximately 45 inches long, premarked 0, 1, and 2 at 20-inch intervals.



Materials: Pennies, pads of small sticky notes (1.5" x 2"), color tiles (1"x 1"; ≈ 0.5 cm thick), tape.

FIGURE 2

To give students a strong visual image of the partitioning process, the teacher has them iterate units of sticky notes.

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After a lively discussion, students agree that all these words contain *dec* and are related to the number 10.

Whole-group work: Creating a decimal racetrack

Using a racetrack as the context, Campos's students now create decimal number lines. Each pair of students gets a number strip that will serve as a representation of a miniature two-mile racetrack (see **fig. 1**). Campos first invites her students to imagine a one-mile interval on the racetrack divided into ten equal parts. Students huddle around their number lines, using their fingers to test length estimates until partners reach consensus on the length of one part. When they share, many students show the interval using their fingers, and others offer such measurement estimates as "about two inches." Campos makes an intentional choice here. Rather than having students measure in standard units, she chooses to have them iterate units to gain a strong visual image of the partitioning process. So she asks, "What do we have in our classroom that is just this size?"

After a bit of brainstorming, she reveals a pad of small sticky notes (1.5 in. × 2 in.); students agree it looks like a good fit (see **fig. 2**). The teacher distributes sticky notes, and students jump right into the task of dividing each mile

FIGURE 3

This game gives students practice reading, writing, and comparing decimal values.

Penny-pushing game directions

Place your penny at the 0 line. Give it a gentle push along the racetrack. Say and write the decimal value closest to where it lands. Take turns with your partner. Do this five times.

FIGURE 4

To strategically play for a higher score requires mathematical thinking.

Penny-pushing scores—TENTHS game

Some values may have more than one letter.

Mark all values MORE THAN ONE AND TWO-TENTHS with an "A."

Mark all values LESS THAN ZERO AND SEVEN-TENTHS with a "B."

Mark all values BETWEEN ONE AND FIVE-TENTHS AND TWO with a "C."

Mark all values BETWEEN FOUR-TENTHS AND ONE AND THREE-TENTHS with a "D."

Give one point for each letter. What is your score?

into ten equal sections by laying the sticky notes end to end on the racetrack. Students record each sticky note interval with a tick mark and then remove the notes.

The next task is to label each position on the racetrack with a decimal value. Campos intentionally connects the decimal notation to students' knowledge of fractions. Tracing a path from the starting line (at zero), past the one-mile line, and stopping at the eighth tick mark beyond it, Campos asks, "How many miles has our runner completed at this point on the track?"

Students quickly name this point as one and eight-tenths mile from the starting line. Campos writes 1 8/10 and 1.8 on the board and explains the connection. She asks, "What do the digits one and eight tell us about this position on the racetrack?"

A student replies, "The one tells us the runner has passed the one-mile mark, and the eight means she's gone eight sticky notes more."

The ability to connect the numeral to positions on the racetrack will become even more important when hundredths are introduced. Students now write decimal values next to all the tick marks on their racetracks.

Partner work: Reading and finding decimal values

At this point, Campos wants to collect some formative assessment data. Can all students read and identify decimal values on this number line? To find out, she calls out some decimal values, and her students point to each one on their racetracks. When she is satisfied that students can accurately find values, she turns this into a partner activity. One partner is the teller; the other is the pointer. They trade roles with each turn, giving each student practice both finding and naming decimal values. This activity affords Campos more formative-assessment opportunities. She makes observations and reminds students to use decimal place-value language rather than using the word *point*.

The Penny Pushing game (see **fig. 3**) will offer students more practice reading and writing decimal values and later comparing them.

Penny Pushing game

While students slide pennies along the racetrack and read the decimal values, Campos circulates among them, providing targeted support. When everyone has completed the task—writing five decimal values based on their penny pushes—Campos reveals the point-scoring list (see **fig. 4**).

Deciding which letters belong with each of their decimal values requires students to make comparisons, and the number line can be used as a reference if needed. After students have calculated their point totals, they are anxious to play again for a higher score. This was Campos's intent. She knows it will take even more mathematical thinking to strategically aim for more points.

Debriefing: Reinforcing the learning

Having a discussion at the end of a lesson is a regular practice in Campos's class. She knows the power of debriefing conversations for solidifying concepts and extending students' thinking. Today, she wants to confront a common misconception (the fraction rule) and preview concepts coming in the next lesson. She writes 1/2 on the board and asks students to point to the half-mile position on their racetracks. As expected, a few students erroneously choose 1.2 because it contains the same digits as the fractional representation. As a result of their subsequent discussion, the students themselves articulate the flaw in this thinking. Campos closes with an exit ticket question previewing the next lesson: “What might be the decimal name for the spot halfway between 1.3 and 1.4?”

Day 2

Objective

Name positions on the number line using decimals (in hundredths); compare and order decimal values.

Launch: What is halfway between 1.3 and 1.4?

Student responses from yesterday’s exit ticket provide a natural starting point for today’s launch. Three common responses to the exit ticket were 1.3 1/2; 1.3.5; and 1.35. Students discuss the logic associated with each suggestion. Campos does not resolve the issue. Instead, she tells students they will return to these after further subdividing their racetracks.

Whole-group work: Creating hundredths on the racetrack

The need to partition the tenths intervals arose previously with the Penny Pushing game. The penny frequently landed between the tenths lines, and these decimal names were approximations. Students readily agree that dividing each of the tenths intervals into ten equal parts would provide more accuracy while maintaining the decimal system of tens.

Visualizing the subdivision process is important, so Campos presents another opportunity to iterate units. Following the same process used with tenths, students first estimate the size of one section within a tenths interval. Campos demonstrates that a color tile held edgewise is a fairly good fit. In fact, they decide to stack and tape ten color tiles into a brick and use this as a convenient measuring device (see **fig. 5**).

Each pair of students makes a color tile brick and divides the interval between 1.3 and 1.4 into ten equal parts. Now they return to the task of naming the spot at the midpoint of this interval. They consider the list of exit ticket responses discussed earlier. Campos explains

The teacher’s goal is to extend students’ thinking beyond hundredths.

FIGURE 5

Students use color tiles to iterate units. They stack ten of them together to use as a measuring device.

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mathematical conventions for the use of the decimal point, which eliminates 1.3.5 from consideration. They agree 1.3 1/2 makes sense but is not a purely decimal representation—which leaves 1.35. Campos confirms that this is the notation used by mathematicians; she uses questions to reinforce students' understanding of the structure and terminology used to name this decimal fraction.

Campos: This is kind of strange. The way this number is written, it contains the number 35. Does thirty-five make sense?

Students: It's thirty-five color tile marks. If we made all the marks from the one-mile line up to here, there would be thirty-five of them. There are ten color tile marks in each sticky note section—ten, twenty, thirty. And this mark is halfway to the fourth one. It's thirty-five little marks past one mile.

Campos: Then how should we read this part of the number? We can't say thirty-five color tile marks! Could it be thirty-five–tenths?

Students: No, thirty-five–hundredths. There would be one hundred color tile marks in all if we marked off the whole section. There are ten in each tenth; ten times ten is one hundred marks in all.

Students now label each tick mark in this interval using the same reasoning. As students work, Campos circulates and addresses the longer-is-larger misconception. She asks,

> Right after the mark for 1.39 is the spot we had already labeled 1.4. What is the logic here? Is 1.4 really larger than 1.39? How can this be if 39 is larger than 4?

She continues to probe until she is sure that students can explain the flaw in the longer-is-larger misconception.

Now students make tick marks to show hundredths along the entire number line, but they do not label them. The rationale here is both practical and pedagogical. On the practical side, it would be time consuming and messy to label all the marks. Pedagogically, there is value in requiring students to use mathematical reasoning to determine decimal names during the next activities.

FIGURE 6

Students played another round of the Penny Pushing game (see **fig. 3**) but record the penny's location to the nearest hundredths and use the hundredths system to score the games.

Penny pushing scores—HUNDREDTHS game

Some values may have more than one letter.

Mark all values MORE THAN ONE AND 35-HUNDREDTHS with an "E."

Mark all values LESS THAN 82-HUNDREDTHS with an "F."

Mark all values BETWEEN 6-TENTHS AND ONE AND 4-TENTHS with a "G."

Mark all values CLOSE TO ONE (NO MORE THAN THREE-TENTHS AWAY) with an "H."

Give one point for each letter. What is your score?

Partner work: Finding and naming hundredths values

As they did yesterday, students practice finding and naming decimal values—now to hundredths—first with the teacher naming values and then in partner pairs, alternating teller and pointer roles and allowing Campos to listen, watch, and ask questions while they work. Students play another round of the Penny Pushing game (see **fig. 3**), but in this version they record the penny's location to the nearest hundredth and score their games using the system for hundredths (see **fig. 6**).

Debriefing

Campos's goal with today's debriefing discussion is to extend students' thinking beyond hundredths and in doing so, to address the longer-is-smaller misconception. The following questions guide the debriefing discussion:

- To get even more accuracy, what could we do?
- If we divided each hundredths interval into ten parts, how many of these parts would be in each mile?
- How could we read and write the decimals for these values?
- Could we divide the intervals even further?

When students have a grasp of thousandths and beyond, Campos addresses the final misconception. She poses this question: "True or false? 0.7851 describes very small parts; therefore, it is less than 0.7."

Student use logic and their racetracks to refute this erroneous thinking.

A strong conceptual anchor

Campos's next steps expand students' understanding of decimal concepts. During subsequent lessons she introduces additional visual models, such as base-ten pieces, grid paper, and rational number wheels, and asks students to make connections between the models. She links decimals with metric measurement in activities that use metersticks and metric scales. Her goal is for students to have robust decimal place-value understanding that is not tied to a single model or context. Teachers like Campos, who have used this lesson to introduce decimals, report that students still confront decimal misconceptions like Tala's, Carrie's, and Xander's, particularly as they tackle tasks that are more abstract or removed from contexts. However, the experience of partitioning the number strip remains a strong conceptual anchor. *Remember the sticky notes and color tiles* is a phrase that helps Campos's students self-correct when tempted to apply flawed thinking to new decimal problems.

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4.NF.C.5
4.NF.C.6
4.NF.C.7

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We will also Storify the conversation for those who cannot join us live.

A new #TCMchat occurs on the second Wednesday of each month at 9:00 p.m. EDT.

REFERENCES

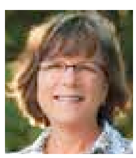
Bamberger, Honi Joyce, Christine Oberdorf, and Karren Schultz-Ferrell. 2010. *Math Misconceptions: Pre-K–Grade 5: From Misunderstanding to Deep Understanding*: Portsmouth, NH: Heinemann.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Martinie, Sherri L. 2007. "Middle School Rational Number Knowledge." PhD diss., p. 747, Curriculum and Instruction, Kansas State University.

Moloney, Kevin, and Kaye Stacey. 1997. "Changes with Age in Students' Conceptions of Decimal Notation." *Mathematics Education Research Journal* 9 (May): 25–38.

Stacey, Kaye, Kevin Moloney, and Vicki Steinle. 2004. *Students' Understanding of Decimals: An Overview* [CD Rom]. Accessed online 6-12-2014. https://extranet.education.unimelb.edu.au/DSME/decimals/SLIMversion/backinfo/refs/molostac.shtml



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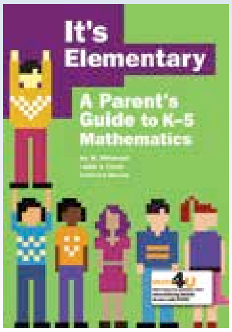

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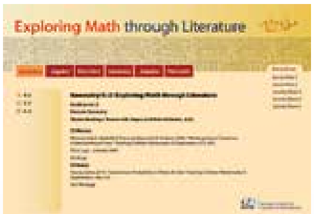

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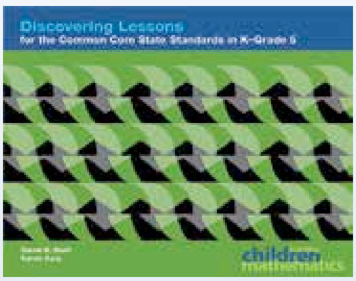

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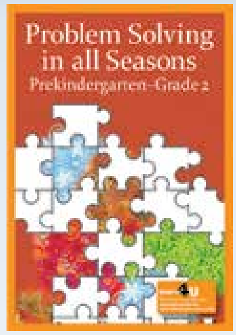

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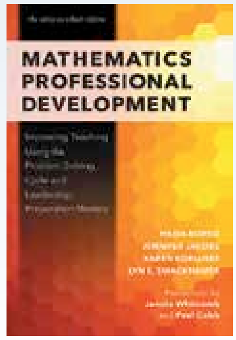

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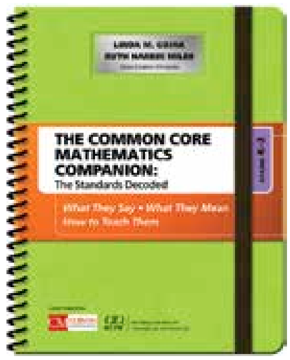

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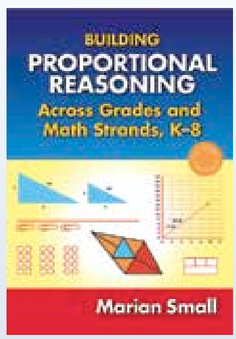

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